Pearson Edexcel

# Mark Scheme (Result)

November 2021

Pearson Edexcel GCE Further Mathematics
Advanced Level in
Further Mathematics Paper 1
9FM0/01

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

# EDEXCEL GCE MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- $\sqcap$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

| Question | Scheme | Marks | |
|---|---|---|---|
| **1(a)** **Way 1** | $\det \mathbf{M} = -4\times -4 - 4\sqrt{3}\times -4\sqrt{3} = ... \Rightarrow k = \sqrt{\det \mathbf{M}} = ...$ | M1 | 3.1a |
| | $k = 8$ | A1 | 1.1b |
| | $\Rightarrow \mathbf{Q} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = ...$ | M1 | 1.1b |
| | $\left(\cos\theta < 0, \sin\theta > 0 \Rightarrow \text{ Quadrant 2 so}\right) \quad \theta = 120°$ | A1 | 1.1b |
| | | **(4)** | |
| **Way 2** | $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = k\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} -4 & -4\sqrt{3} \\ 4\sqrt{3} & -4 \end{pmatrix}$ | M1 | 3.1a |
| | Achieves both the equations $k\cos\theta = -4$ and $k\sin\theta = 4\sqrt{3}$ | A1 | 1.1b |
| | $\frac{k\sin\theta}{k\cos\theta} = \frac{4\sqrt{3}}{-4} \Rightarrow \tan\theta = -\sqrt{3} \Rightarrow \theta = ...$ | M1 | 1.1b |
| | $\theta = 120^\circ$ and $k = 8$ | A1 | 1.1b |
| | | **(4)** | |
| **(b)** | Area of $S'$ = area of $S \times k^2$ (The area of the square $S = 2a^2$) | M1 | 1.1b |
| | Area of $S'$ = $128a^2$ | A1ft | 2.2a |
| | | **(2)** | |
| | | | **(6 marks)** |

**Notes:**

**(a) Way 1**

**M1:** A full method to find $k$ such as attempting the square root of the determinant of **M**. It is immediately deducible so the method may be implied by $k = 8$.

**A1:** $k = 8$

**M1:** A full method to find a value of $\theta$ using their $k$, no need to justify quadrant. Only one equation needed for this mark. Allow if a radians answer is given. May be implied by a correct angle.

**A1:** Correct angle in degrees.

**Way 2**

**M1:** Multiplies the correct matrix representing transformation $Q$ by the matrix representing transformation $P$ and sets equal to matrix **M**. Allow for the matrices either way round as the transformations commute. No need to see the identity matrix, just multiplying through by $k$ is sufficient.

**A1:** Both correct equations. Note that if a correct value of $k$ is found, this A is scored under Way 1.

**M1:** Solves their simultaneous equations to find a value for $\theta$ (or $k$)

**A1:** $\theta = 120^\circ$ and $k = 8$

**(b)**

**M1:** Complete method to find the area of $S'$ : 'their $k^2$' $\times$ 'their $2a^2$'. Must be an attempt at the area of $S$ but it need not be correct.

**A1ft:** Deduces the correct area for $S'$, follow through their value of $k$

| Question | Scheme | Marks | |
|---|---|---|---|
| 2 (a) | $\cos^2\frac{x}{3}=\left(1-\frac{\left(\frac{x}{3}\right)^2}{2}+\frac{\left(\frac{x}{3}\right)^4}{24}-\ldots\right)^2=\ldots$ or $\left(1-\frac{x^2}{18}+\frac{x^4}{1944}-\ldots\right)^2=\ldots$ or $\frac{1}{2}\left(1\pm\cos\frac{2x}{3}\right)=\frac{1}{2}\left(1\pm\left(1-\frac{1}{2}\left(\frac{2x}{3}\right)^2+\frac{1}{4!}\left(\frac{2x}{3}\right)^4-\right)\ldots\right)$ | M1 | 2.2a |
| | $=1-\frac{x^2}{9}+\frac{1}{243}x^4$ | A1 | 1.1b |
| | | **(2)** | |
| (b) | $\int\frac{1-\frac{x^2}{9}+\frac{1}{243}x^4}{x}=\int\frac{1}{x}-\frac{x}{9}+\frac{1}{243}x^3=A\ln x+Bx^2+Cx^4$ where $A$, $B$ and $C\neq 0$ | M1 | 3.1a |
| | $\ln x-\frac{x^2}{18}+\frac{1}{972}x^4$ | A1ft | 1.1b |
| | = awrt 0.98295 | A1 | 2.2a |
| | | **(3)** | |
| (c) | Calculator = awrt 0.98280 | B1 | 1.1b |
| | | **(1)** | |
| (d) | E.g. the approximation is correct to 3 d.p. | B1 | 3.2b |
| | | **(1)** | |
| | | **(7 marks)** | |

**Notes:**

**(a)**

**M1:** Deduces the required series by using the Maclaurin series for cos $x$, replacing $x$ with $\frac{x}{3}$ and squares, or first applying the double angle identity (allow sign error) and then applying the series for cos $x$ with $\frac{2x}{3}$. Attempts at finding from differentiation score M0 as the cosine series is required.

**A1:** Correct series

**(b)**

**M1:** Divides their series in part (a) by $x$ and integrates to the form $A\ln x+Bx^2+Cx^4$

**A1ft:** Correct integration, follow through on their coefficients and need not be simplified.

**A1:** Deduces the definite integral awrt 0.98295

**(c)**

**B1:** Correct value.

**(d)**

**B1:** Makes a quantitative statement about the accuracy, so e.g. how many decimal places or significant figures it is correct to, or calculates a percentage accuracy to deduce it is reasonable. Do not accept just "underestimate" or similar without quantitative evidence. Allow for a reasonable comment as long as (b) is correct to at least 2 s.f. but (c) must be the correct value.

| Question | Scheme | Marks | |
|---|---|---|---|
| **3** | $w=4x-1 \Rightarrow x=\dfrac{w+1}{4}$ | B1 | 3.1a |
| | $a\left(\dfrac{w+1}{4}\right)^3+b\left(\dfrac{w+1}{4}\right)^2-19\left(\dfrac{w+1}{4}\right)-b\ (=0)$ or <br> $(4x-1)^3-9(4x-1)^2-97(4x-1)+c\ (=0)$ | M1 | 3.1a |
| | $aw^3+(3a+4b)w^2+(3a+8b-304)w+(a-60b-304)=0$ <br> or $64x^3 - 192x^2 - 304x + 87 + c = 0$ | M1 | 1.1b |
| | Divides by $a$ and equates the coefficients of $w^2$ and $w$ <br> $\dfrac{3a+4b}{a}=-9 \quad \dfrac{3a+8b-304}{a}=-97$ <br> and solves simultaneously to find a value for $a$ or a value for $b$ <br> **<u>Note:</u>** $12a+4b=0$ and $100a+8b=304$ <br> or <br> Divides through by '16' leading to values of $a$ and $b$ <br> $4x^3 - 12x^2 - 19x + \dfrac{87+c}{19} = 0$ | M1 | 3.1a |
| | $c=\dfrac{a-60b-304}{a}=\ldots$ or $\dfrac{87+c}{19}=12\ \text{Þ}\ c=\ldots$ | M1 | 1.1b |
| | $a=4 \quad b=-12 \quad c=105$ | A1 | 1.1b |
| | | **(6)** | |
| | | | **(6 marks)** |

**Notes:**

**B1:** Selects the method of making a connection between $x$ and $w$ by writing $w=4x-1$ or $x=\dfrac{w+1}{4}$

**M1:** Applies the process of substituting their $x=\dfrac{w+1}{4}$ into $ax^3+bx^2-19x-b=0$ or $w=4x-1$ into $w^3-9w^2-97w+c=0$. Must be substitution of the correct variable into the opposing equation but may be scored if the initial linear equation is incorrect (e.g. $x=4w-1$ into the first equation). Note that the " = 0 " can be missing for this mark.

**M1:** Expands the brackets and collects terms in their equation (in $x$ or $w$). Note that the " = 0 " can be missing for this mark.

**M1:** A complete method for finding a value for $a$ or $b$. See scheme, it involves dividing through by an appropriate factor for their equation to balance the $w^3$ or $-19x$ terms, then equating other coefficients and solving equations if necessary.

**M1:** A complete method for finding a value for $c$. They must have divided through by an appropriate factor as per the previous M before attempting to compare the constant coefficient (and use their $a$ and $b$ if appropriate).

**A1:** $a=4 \quad b=-12 \quad c=105$

| | | | |
|---|---|---|---|
| | **Alternative**<br>At least two of $\alpha+\beta+\gamma=-\frac{b}{a}$ $\quad \alpha\beta+\alpha\gamma+\beta\gamma=-\frac{19}{a}$ $\quad \alpha\beta\gamma=\frac{b}{a}$ | B1 | 3.1a |
| | New sum $=4(\alpha+\beta+\gamma)-3=9 \Rightarrow 4\left(-\frac{b}{a}\right)-3=9 \Rightarrow b=-3a$ | M1 | 3.1a |
| | New pair sum$=16(\alpha\beta+\alpha\gamma+\beta\gamma)-8(\alpha+\beta+\gamma)+3=-97$<br>$\Rightarrow 16\left(-\frac{19}{a}\right)-8\left(-\frac{b}{a}\right)+3=-97$ | M1 | 1.1b |
| | $\Rightarrow 16\left(-\frac{19}{a}\right)-8(3)+3=-97 \Rightarrow a=\ldots$ | M1 | 3.1a |
| | New product $64(\alpha\beta\gamma)-16(\alpha\beta+\alpha\gamma+\beta\gamma)+4(\alpha+\beta+\gamma)-1=-c$<br>$\Rightarrow 64\left(\frac{b}{a}\right)-16\left(-\frac{19}{a}\right)+4(3)-1=-c \Rightarrow c=\ldots$ | M1 | 1.1b |
| | $a=4 \quad b=-12 \quad c=105$ | A1 | 1.1b |
| | | **(6)** | |

**Alternative Notes**

**B1:** Selects the method of giving at least two correct equations containing $\alpha$, $\beta$ and $\gamma$

**M1:** Applies the process of finding the new sum to generate an equation in *a* and *b*. Must be substituting in the correct places.

**M1:** Attempts the new pair sum to generate another equation connecting *a* and *b*. Must be substituting in the correct places.

**M1:** Solves their equations to find a value for *a* or *b*.

**M1:** Uses the new product with their values to find values for *a*, *b* and *c*

**A1:** $a=4 \quad b=-12 \quad c=105$

| Question | Scheme | Marks | |
|---|---|---|---|
| **4(i) (a)** | It is possible as the number of columns of matrix **A** matches the number of rows of matrix **B**. | B1 | 2.4 |
| **(b)** | It is not possible as matrix **A** and matrix **B** have different dimensions o.e. different number of columns | B1 | 2.4 |
| | | **(2)** | |
| **(ii) (a)** | $\lambda = 5$ | B1 | 2.2a |
| | $a = 1,\ b = 2$ | B1 | 2.2a |
| **(b)** | Inverse matrix $= \frac{1}{5}\begin{pmatrix} 0 & 5 & 0 \\ 2 & 12 & -1 \\ -1 & -11 & 3 \end{pmatrix}$ | B1 ft | 3.1a |
| | | **(3)** | |
| **(iii)** | A complete method to find the determinant of the matrix and set equal to zero. | M1 | 1.1b |
| | Determinant = $1(\sin\theta\sin 2\theta - \cos\theta\cos 2\theta) - 1(0) + 1(0) = 0$ | A1 | 1.1b |
| | Uses compound angle formula to achieve $\cos 3\theta = 0$ leading to $\theta = ...$<br>or<br>use of $\sin 2q = 2\sin q\cos q$ and $\cos 2q = 1 - 2\sin^2 q$ (e.g. to achieve $\cos q(4\sin^2 q - 1) = 0$) leading to $\theta = ...$<br>or<br>use of $\sin 2q = 2\sin q\cos q$ and $\cos 2q = 2\cos^2 q - 1$ (e.g. to achieve $4\cos^3 q - 3\cos q = 0$) leading to $\theta = ...$ | M1 | 3.1a |
| | $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ | A1 | 1.1b |
| | | **(4)** | |
| | | | **(9 marks)** |

**Notes:**

**(i)(a)**
**B1:** Comments that the number of columns of matrix **A** (2) equals the number of rows of matrix **B** (2) therefore it is possible. Accept other terminology that is clear in intent e.g. "length of **A**" and "height of **B**"
**(b)**
**B1:** Comments that matrix **A** and matrix **B** have different dimensions therefore it is not possible.

**(ii)(a)**
**B1:** Deduces the correct value for $\lambda = 5$
**B1:** Deduces the correct values for $a$ and $b$
**(b)**
**B1ft**: Identifies and applies a correct method find the inverse matrix. May multiply from the given equation, in which case follow through on their value of lambda. Alternatively, award for a correct matrix found by calculator or long hand having found $a$ and $b$ and using these values in the matrix.

**(iii)**
**M1:** A complete method to find the determinant of the matrix and sets it equal to 0
**A1:** Correct equation
**M1:** Uses appropriate correct trig identities to solve the equation and finds a value for $q$
**A1**: All three correct values $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ and no others in the range.

| Question | Scheme | Marks | |
|---|---|---|---|
| **5(i)** | $\int 2e^{-\frac{1}{2}x}\,dx = -4e^{-\frac{1}{2}x}$ | B1 | 1.1b |
| | $\int_1^{\infty} 2e^{-\frac{1}{2}x}\,dx = \lim_{t\to\infty}\left[\left(-4e^{-\frac{1}{2}t}\right)-\left(-4e^{-\frac{1}{2}}\right)\right]$ | M1 | 2.1 |
| | $= 4e^{-\frac{1}{2}}$ | A1 | 1.1b |
| | | **(3)** | |
| **(ii)(a)** | Mean temperature $= \frac{1}{24}\int_0^{24}\left(8-5\sin\left(\frac{\pi}{12}t\right)-\cos\left(\frac{\pi}{6}t\right)\right)dt$ | B1 | 1.2 |
| | $= \frac{1}{24}\left[\left(8t+\frac{60}{\pi}\cos\left(\frac{\pi}{12}t\right)-\frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\right)\right]_0^{24} = \frac{1}{24}[...]$ | M1 | 1.1b |
| | $= \frac{1}{24}\left[\left(8(24)+\frac{60}{\pi}-\frac{6}{\pi}\times 0\right)-\left(\frac{60}{\pi}\right)\right] = 8$ * cso | A1* | 2.1 |
| | | **(3)** | |
| **(ii)(b)** | E.g. increase the value of the constant 8 / adapt the constant 8 to a function which takes values greater than 8. | B1 | 3.5c |
| | | **(1)** | |
| | | **(7 marks)** | |

**Notes:**

**(i)**

**B1:** Correct integration.

**M1**: Attempt to integrate to a form $\lambda e^{-\frac{1}{2}x}$ where $\lambda \neq 2$, and applies correct limits with some consideration of the infinite limit given (e.g. with the limit statement). Only allow with $\infty$ used as the limit if subsequent work shows the term is zero.

**A1:** Correct value

**(ii)(a)**

**B1:** Recalls the correct formula for finding the mean value of a function. You may see the division by "24" only at the end. No integration is necessary, just a correct statement with an integral.

**M1:** Integrates to a form $\alpha t+\beta\cos\left(\frac{\pi}{12}t\right)+\delta\sin\left(\frac{\pi}{6}t\right)$ and uses the limits of 0 and 24 (the correct way around). If no explicit substitution is seen, accept any value following the integral as an attempt. Answers from a calculator with no correct integral seen score M0 as the question requires calculus to be used.

**A1*cso:** Achieves 8 with no errors seen following a full attempt at the substitution. Must have seen some evidence of the limits used, minimum required for substitution is $\left[\left(8(24)+\frac{60}{\pi}\right)-\left(\frac{60}{\pi}\right)\right]$.

**(ii)(b)**

**B1:** Accept any reasonable adaptation to the equation that will increase the mean value. E.g. as in scheme, or introduce another positive term, or decrease the constant 5 etc. It must be clear which constant they are referring to in their reason, not just "increase the constant".

| Question | Scheme | Marks | |
|---|---|---|---|
| **6(a)** | $5k(13.6)+2k(0)+17(-20)=0 \Rightarrow k=...$ | M1 | 3.3 |
| | $k=5$ | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | Solves their $25m^2+10m+17=0 \Rightarrow m=...$ | M1 | 3.1b |
| | $m=-0.2\pm 0.8\mathrm{i}$ | A1 | 1.1b |
| | $x=\mathrm{e}^{-0.2t}(A\cos 0.8t+B\sin 0.8t)$ | A1ft | 1.1b |
| | $t=0,\ x=-20 \Rightarrow A=...\ (=-20)$ | M1 | 3.4 |
| | $\frac{\mathrm{d}x}{\mathrm{d}t}=-0.2\mathrm{e}^{-0.2t}(A\cos 0.8t+B\sin 0.8t)$ $+\mathrm{e}^{-0.2t}(-0.8A\sin 0.8t+0.8B\cos 0.8t)$ | M1 | 1.1b |
| | $t=0\ \frac{\mathrm{d}x}{\mathrm{d}t}=0 \Rightarrow -0.2A+0.8B=0 \Rightarrow B=...\ (=-5)$ | dM1 | 3.4 |
| | $x=\mathrm{e}^{-0.2t}(-20\cos 0.8t-5\sin 0.8t)$ o.e. | A1 | 1.1b |
| | | **(7)** | |
| **(c)** | Vertical height $=30+\left[\mathrm{e}^{-0.2\times 15}(-20\cos(0.8\times 15)-5\sin(0.8\times 15))\right]$ | M1 | 3.4 |
| | Vertical height = awrt 29.3 m | A1 | 2.2b |
| | | **(2)** | |
| **(d)** | For example<br>It is unlikely that the rope will remain taut<br>The model predicts the tourist will continue to move up and down, (but in fact they will lose momentum)<br>The tourist is modelled as a particle | B1 | 3.5b |
| | | **(1)** | |
| | | **(12 marks)** | |

**Notes:**

**(a)**

**M1:** Substitutes $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=13.6$ $\frac{\mathrm{d}x}{\mathrm{d}t}=0$ and $x=-20$ into the differential equation to find a value for $k$.
Allow if there are sign slips but must be attempting the values in the correct places.
**A1:** Correct value $k=5$

**(b)**
**M1:** Forms and solves the auxiliary equation.
**A1:** Correct solution to the auxiliary equation (not follow through).
**A1ft:** Correct complementary function for their solutions to their auxiliary equation. (Follow through on distinct real, repeated or complex roots.)

| |
|---|
| **M1:** Uses the information from the model $t=0\ x=-20$ to find a constant or equation linkin constants in their equation.<br>**M1:** Differentiates an expression of the form $e^{kt}\left(A\cos\lambda_1 t + B\sin\lambda_2 t\right)$ using the product rule to find an expression for the velocity.<br>**dM1:** Uses the information from the model, $t=0\ \frac{dx}{dt}=0$ to find and solve another equation for the constants.<br>**A1:** Correct equation for displacement. |
| **(c)**<br>**M1:** Finds the height above the river by finding the displacement after 15 seconds and adding 30<br>**A1**: Vertical height = awrt 29.3 m |
| **(d)**<br>**B1:** Any suitable comment relating to the given model or the outcomes of it. See scheme for examples. Do not accept just "air resistance has not been considered" as the question does not say this was ignored. However, if a valid consequence of what including air resistance would mean to the model, then the mark may be awarded. |

| Question | Scheme | Marks | |
|---|---|---|---|
| 7(a) | $\begin{pmatrix}-1\\2\\1\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}=-2+6-4=0$ and $\begin{pmatrix}2\\0\\1\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}=4+0-4=0$ <br> Alt: $\begin{pmatrix}-1\\2\\1\end{pmatrix}\times\begin{pmatrix}2\\0\\1\end{pmatrix}=\begin{pmatrix}2\times1-1\times0\\-(-1\times1-1\times2)\\-1\times0-2\times2\end{pmatrix}=\ldots$ | M1 | 1.1b |
| | As 2**i** + 3**j** – 4**k** is perpendicular to both direction vectors (two non-parallel vectors) of $\Pi$ then it must be perpendicular to $\Pi$ | A1 | 2.2a |
| | | **(2)** | |
| (b) | $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}=\begin{pmatrix}3\\3\\2\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}\Rightarrow\ldots$ | M1 | 1.1a |
| | $2x+3y-4z=7$ | A1 | 2.2a |
| | | **(2)** | |
| (c) | $\dfrac{\lvert 2(4+t)+3(-5+6t)-4(2-3t)-7\rvert}{\sqrt{2^2+3^2+(-4)^2}}=2\sqrt{29}\Rightarrow t=\ldots$ | M1 | 3.1a |
| | $t=-\dfrac{9}{8}$ and $t=\dfrac{5}{2}$ | A1 | 1.1b |
| | $\boldsymbol{r}=\begin{pmatrix}4\\-5\\2\end{pmatrix}-\dfrac{9}{8}\begin{pmatrix}1\\6\\-3\end{pmatrix}=\ldots$ or $\boldsymbol{r}=\begin{pmatrix}4\\-5\\2\end{pmatrix}+\dfrac{5}{2}\begin{pmatrix}1\\6\\-3\end{pmatrix}=\ldots$ | M1 | 1.1b |
| | $\left(\dfrac{23}{8},-\dfrac{47}{4},\dfrac{43}{8}\right)$ and $\left(\dfrac{13}{2},10,-\dfrac{11}{2}\right)$ | A1 | 2.2a |
| | | **(4)** | |
| | | | **(8 marks)** |

**Notes:**

**(a)**
**M1:** Attempts the scalar product of each direction vector and the vector 2**i** + 3**j** – 4**k.** Some numerical calculation is required, just "= 0" is insufficient. Alternatively, attempts the cross product (allow sign slips) with the two direction vectors.
**A1:** Shows that both scalar products = 0 (minimum $-2+6-4=0$ and $4-4=0$) and makes a minimal conclusion with no erroneous statements. If using cross product, the calculation must be correct, and a minimal conclusion given with no erroneous statements.

**(b)**

**M1:** Applies $\begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} . \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \Rightarrow \ldots$

**A1:** $2x+3y-4z=7$

**(c)**

**M1:** A fully correct method for finding a value of $t$. Other methods are possible, but must be valid and lead to a value of $t$. Examples of other methods:

- $2\sqrt{29} = \pm\left(\dfrac{2(4+t)+3(-5+6t)-4(2-3t)}{\sqrt{2^2+3^2+(-4)^2}} - \dfrac{7}{\sqrt{29}}\right)$ using plane parallel to $\Pi$ through origin and shortest distance from plane to origin.
- $2(4+t)+3(-5+6t)-4(2-3t)=7 \Rightarrow t=t_i$ ($t$ at intersection of line and plane) and $\sin\theta = \dfrac{(2,3,-4)^T.(1,6,-3)^T}{\sqrt{29}\sqrt{46}}$ (sine of angle between line and plane) followed by $\sin\theta = \dfrac{2\sqrt{29}}{k\sqrt{46}} \Rightarrow k = \ldots \Rightarrow t = t_i \pm k$

**A1:** Correct values for $t$. Both are required.

**M1:** Uses a value of $t$ to find a set of coordinates for $A$.

**A1:** Both correct sets of coordinates for $A$.

| Question | Scheme | | Marks | |
|---|---|---|---|---|
| **8(a)** | Volume of paint = 30 litres therefore<br>Rate of paint out $= 3 \times \dfrac{r}{30}$ litres per second | | M1 | 3.3 |
| | $\dfrac{dr}{dt} = 2 - \dfrac{r}{10}$ | | A1 | 1.1b |
| | | | **(2)** | |
| **(b)** | Rearranges $\dfrac{dr}{dt} + \dfrac{r}{10} = 2$ and attempts integrating factor<br>IF = $e^{\int \frac{1}{10} dt} = \ldots$ | Separates the variables<br>$\int \dfrac{1}{20-r} dr = \dfrac{1}{10} dt$<br>$\Rightarrow \ldots$ | M1 | 3.1a |
| | $re^{\frac{t}{10}} = \int 2e^{\frac{t}{10}} dt \Rightarrow re^{\frac{t}{10}} = \lambda e^{\frac{t}{10}} (+c)$ | Integrates to the form<br>$\lambda \ln(20-r) = \dfrac{1}{10} t (+c)$ | M1 | 1.1b |
| | $re^{\frac{t}{10}} = 20e^{\frac{t}{10}} + c$ | $-\ln(20-r) = \dfrac{1}{10} t + c$ | A1ft | 1.1b |
| | $t = 0, r = 10 \Rightarrow c = \ldots$ | | M1 | 3.4 |
| | $r = \dfrac{20e^{\frac{t}{10}} - 10}{e^{\frac{t}{10}}} = 15$ rearranges to achieve $e^{\frac{t}{10}} = \alpha$ and solves to find a value for $t$<br>or<br>$r = 20 - 10e^{-\frac{t}{10}} = 15$ rearranges to achieve $e^{-\frac{t}{10}} = \beta$ and solves to find a value for $t$ | $-\ln(20-15) = \dfrac{1}{10} t - \ln 10$<br>Leading to a value for $t$ | M1 | 3.4 |
| | $t =$ awrt 7 seconds | | A1 | 2.2b |
| | | | **(6)** | |
| **(c)** | The model predicts 7 seconds but it actually takes 9 seconds so (over) 2 seconds out (over 20%), therefore it is not a good model | | B1ft | 3.5a |
| | | | **(1)** | |
| | | | **(9 marks)** | |

**Notes:**

**(a)**

**M1:** Clearly identifies that Rate of paint out $= 3 \times \dfrac{r}{\text{their volume}}$. It is a "show that" question so there must be clearly reasoning. Just answer with no reasoning scores M0.

**A1:** Puts all the components together to form the correct differential equation.

**(b)**

**M1:** Identifies as a first order differential equation and finds the integrating factor or separates the variables and integrates. Allow if there are sign slips in rearranging (e.g. to $\frac{dr}{dt} - \frac{r}{10} = 2$) or in the integrating factor and allow with their value for $a$ or with $a$ as an unknown.

**M1:** Multiplies through by the IF and attempts to integrate or integrates to the form $\lambda \ln(2a - r) = \frac{1}{a}t + c$ oe

**A1ft:** Correct integration, including constant of integration. Follow through on their value of $a$, but not sign slips from rearrangement. So allow for $re^{\frac{t}{a}} = 2a\,e^{\frac{t}{a}} + c$ or $-\ln(2a - r) = \frac{1}{a}t + c$ oe with $a$ or their $a$.

**M1:** Uses the initial conditions to find the constant of integration. Must see substitution or can be implied by the correct value for their equation. Allow for finding in terms of $a$ if separation of variables used.

**M1:** Sets $r = 15$, achieves $e^{\frac{t}{10}} = \alpha > 0$ or $e^{-\frac{t}{10}} = \beta > 0$ as appropriate and solves to find a value for $t$. Separates the variable method sets $r = 15$ and rearranges to find a value for $t$. **Note:** For this mark a value of $a$ is needed, but need not be the correct one.

**A1cso:** $t =$ awrt 7 seconds from fully correct work.

**(c)**

**B1ft:** See scheme, follow through on their answer to part (b). Accept any reasonable comparative comment but must have a reason, not just a statement of good or not good. So e.g. look for finding the difference between their answer and 9, or the percentage difference. If their answer is close to 9, then accept a conclusion of being a good model if a suitable reason is given. May substitute 9 into their equation and obtain a value to compare with 15 and make a similar conclusion.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 9(a) | $\displaystyle\int \frac{x^2}{\sqrt{x^2-1}}\,\mathrm{d}x \rightarrow \int \mathrm{f}(u)\,\mathrm{d}u$<br>Uses the substitution $x=\cosh u$ fully to achieve an integral in terms of $u$ only, including replacing the $\mathrm{d}x$ | M1 | 3.1a |
| | $\displaystyle\int \frac{\cosh^2 u}{\sqrt{\cosh^2 u-1}}\sinh u\,(\mathrm{d}u)$ | A1 | 1.1b |
| | Uses correct identities<br>$\cosh^2 u-1=\sinh^2 u$ and $\cosh 2u=2\cosh^2 u-1$<br>to achieve an integral of the form<br>$A\int(\cosh 2u\pm1)\,\mathrm{d}u \qquad A>0$ | M1 | 3.1a |
| | Integrates to achieve $A\left(\pm\frac{1}{2}\sinh 2u\pm u\right)(+c) \qquad A>0$ | M1 | 1.1b |
| | Uses the identity $\sinh 2u=2\sinh u\cosh u$ and $\cosh^2 u-1=\sinh^2 u$<br>$\rightarrow \sinh 2u=2x\sqrt{x^2-1}$ | M1 | 2.1 |
| | $\frac{1}{2}\left[x\sqrt{x^2-1}+\operatorname{ar}\cosh x\right]+k$ * cso | A1* | 1.1b |
| | | **(6)** | |
| (b) | Uses integration by parts the correct way around to achieve<br>$\displaystyle\int\frac{4}{15}x\,\text{arcosh}x\mathrm{d}x=Px^2\text{arcosh}x-Q\int\frac{x^2}{\sqrt{x^2-1}}\mathrm{d}x$ | M1 | 2.1 |
| | $\displaystyle=\frac{4}{15}\left(\frac{1}{2}x^2\text{arcosh}x-\frac{1}{2}\int\frac{x^2}{\sqrt{x^2-1}}\mathrm{d}x\right)$ | A1 | 1.1b |
| | $\displaystyle=\frac{4}{15}\left(\frac{1}{2}x^2\text{arcosh}x-\frac{1}{2}\left(\frac{1}{2}\left[x\sqrt{x^2-1}+\text{arcosh}x\right]\right)\right)$ | B1ft | 2.2a |
| | Uses the limits $x=1$ and $x=3$ the correct way around and subtracts<br>$\displaystyle=\frac{4}{15}\left(\frac{1}{2}(3)^2\text{arcosh}3-\frac{1}{2}\left(\frac{1}{2}\left[3\sqrt{(3)^2-1}+\text{arcosh}3\right]\right)\right)-\frac{4}{15}(0)$ | dM1 | 1.1b |
| | $\displaystyle=\frac{4}{15}\left(\frac{9}{2}\ln\left(3+\sqrt{8}\right)-\frac{3\sqrt{8}}{4}-\frac{1}{4}\ln\left(3+\sqrt{8}\right)\right)$<br>$\displaystyle=\frac{1}{15}\left[17\ln\left(3+2\sqrt{2}\right)-6\sqrt{2}\right]$* | A1* | 1.1b |
| | | **(5)** | |
| | | **(11 marks)** | |
| | | | |

**Notes:**

**(a)**

**M1:** Uses the substitution $x = \cosh u$ fully to achieve an integral in terms of $u$ only. Must have replaced the d$x$ but allow if the d$u$ is missing.

**A1:** Correct integral in terms of $u$. (Allow if the d$u$ is missing.)

**M1:** Uses correct identities $\cosh^2 u - 1 = \sinh^2 u$ and $\cosh 2u = 2\cosh^2 u - 1$ to achieve an integrand of the required form

**M1:** Integrates to achieve the correct form, may be sign errors.

**M1:** Uses the identities $\sinh 2u = 2\sinh u \cosh u$ and $\cosh^2 u - 1 = \sinh^2 u$ to attempt to find $\sinh 2u$ in terms of $x$. If using exponentials there must be a full and complete method to attempt the correct form.

**A1*:** Achieves the printed answer with no errors seen, cso

**NB** attempts at integration by parts are not likely to make progress – to do so would need to split the integrand as $x\dfrac{x}{\sqrt{x^2-1}}$. If you see any attempts that you feel merit credit, use review.

**(b)**

**M1:** Uses integration by parts the correct way around to achieve the required form.

**A1:** Correct integration by parts

**B1ft:** Deduces the integral by using the result from part (a). Follow through on their '$uv$'

**dM1:** Dependent on previous method mark. Uses the limits $x = 1$ and $x = 3$ the correct way around and subtracts

**A1*cso:** Achieves the printed answer with at least one intermediate step showing the evaluation of the arcosh 3, and no errors seen.